Mathematics

31. If $I_m = \int_0^{\pi/4} (\tan x)^m dx$, then $I_3 + I_5 + I_7 + I_9$ equals:

A. $\frac{3}{8}$. B. $\frac{3}{7}$. C. $\frac{2}{5}$. D. $\frac{4}{9}$.

32. Which is the largest number in the following sequence?

$$1^{\frac{1}{\sqrt{1}}}, 3^{\frac{1}{\sqrt{3}}}, 5^{\frac{1}{\sqrt{5}}}, \cdots (2n+1)^{\frac{1}{\sqrt{2n+1}}}, \cdots$$

A. $3^{\frac{1}{\sqrt{3}}}$.

B. $5^{\frac{1}{\sqrt{5}}}$.

C. $7^{\frac{1}{\sqrt{7}}}$.

D. The sequence is unbounded.

33. Assuming that the interchange of limit and integration is permissible, the value of

$$\lim_{n\to\infty} \int_0^1 \frac{nx^{n-1}}{1+x}\, dx \quad ; 0 < x < 1$$

is:

A. 0. B. $\frac{1}{2}$. C. 1. D. ∞.

34. Let $f : \mathbb{R} \to \mathbb{R}$ be a function such that $|f(x) - f(y)| \leq 6|x - y|^2$ for all $x, y \in \mathbb{R}$. If $f(3) = 6$ then $f(6)$ equals:

A. 6. B. 9. C. 12. D. 18.

35. Let A_n be the area bounded by the curves $y = x$ and $y = nx^2$ in the first quadrant. Then the value of $\sum_{n=1}^{5} \frac{1}{A_n}$ is:

A. 110. B. 220. C. 330. D. 440.

36. In how many ways can four distinguishable pieces be placed on an 8×8 chessboard so that no two pieces are in the same row or column?

A. $\frac{8!}{4!}$. B. $\frac{(8!)^2}{(4!)}$. C. $\frac{(8!)}{(4!)^2}$. D. $\frac{(8!)^2}{(4!)^2}$.

37. A and B are playing a game by alternately rolling a die, with A starting first. Each player's score is the number obtained on his last roll. The game ends when the sum of scores of the two players is 7, and the last player to roll the die wins. What is the probability that A wins the game?

A. $\frac{11}{36}$. B. $\frac{5}{11}$. C. $\frac{17}{36}$. D. $\frac{6}{11}$.

38. The binomial coefficients $\binom{n}{r}, \binom{n}{r+1}, \binom{n}{r+2}; 0 \leq r \leq n-2$:

A. Can be in A.P. or in G.P.

B. Can be in A.P. but never in G.P.

C. Can be in G.P. but never in A.P.

D. Can never be in A.P. or G.P.

39. The sum of the infinite series $\cot^{-1} 2 + \cot^{-1} 8 + \cot^{-1} 18 + \cdots + \cot^{-1}(2n^2) + \cdots$ is:

A. $\frac{\pi}{3}$. B. $\frac{\pi}{4}$. C. $\frac{\pi}{6}$. D. $\frac{\pi}{8}$.

40. The number of integer values of k for which the equation $7\cos\theta + 5\sin\theta = 2k + 1$ has real solutions is:

A. 6. B. 8. C. 10. D. 12.

41. The complex solutions of $(z+i)^{2011} = z^{2011}$ lie on:

A. A circle.

B. An ellipse.

C. A hyperbola.

D. A straight line.

42. How many 2×2 matrices A satisfy both $A^3 = I_2$ and $A^2 = A^t$, where I_2 denotes the 2×2 identity matrix and A^t denotes the transpose of A?

A. 0. B. 1. C. 2. D. 3.

43. Let C be the circle that touches the X-axis and whose centre coincides with the circumcentre of the triangle defined by $4|x| + 3y = 12;\ y \geq 0$. How many points with both co-ordinates integers are there in the interior of C?

A. 0. B. 1. C. 2. D. 3.

44. Let P and Q be the centres of the circles that pass through $(0, 2)$ and $(0, 8)$ and touch the X-axis. Then the equation of the ellipse with P and Q as foci and touching the X-axis is:

A. $\frac{x^2}{41} + \frac{(y-5)^2}{25} = 1$.

B. $\frac{x^2}{16} + \frac{(y-5)^2}{25} = 1$.

C. $\frac{(x-5)^2}{41} + \frac{y^2}{25} = 1$.

D. $\frac{(x-5)^2}{16} + \frac{y)^2}{25} = 1$.

45. Let $f : \mathbb{R} \to \mathbb{R}$ be a function such that $f(x + y) + f(x - y) = f(xy)$ for all $x, y \in \mathbb{R}$. Then f is:

    A. Strictly increasing.

    B. Strictly decreasing.

    C. Identically zero.

    D. Constant but not necessarily zero.