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IIT-JEE

Previous Year

Paper - I

"2008"

(Mathematics)

Mathematics

PART – I

SECTION – I

Straight Objective Type

This section contains 6 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

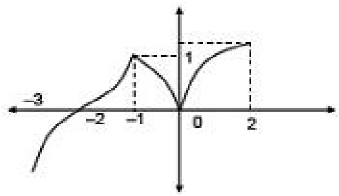
1. Let a and b be non-zero real numbers. Then, the equation $(ax^2 + by^2 + c)(x^2 - 5xy + 6y^2) = 0$ represents
(A) four straight lines, when c = 0 and a, b are of the same sign
(B) two straight lines and a circle, when a = b, and c is of sign opposite to that of a
(C) two straight lines and a hyperbola, when a and b are of the same sign and c is of sign opposite to that of a
(D) a circle and an ellipse, when a and b are of the same sign and c is of sign opposite to that of a

Sol. (B)
$(ax^2 + by^2 + c)(x^2 - 5xy + 6y^2) = 0$
$\Rightarrow ax^2 + by^2 + c = 0$ or $x^2 - 5xy + 6y^2 = 0$
$\Rightarrow x^2 + y^2 = \left(-\frac{c}{a}\right)$ iff $a = b$, $x - 2y = 0$ and $x - 3y = 0$
Hence the given equation represents two straight lines and a circle, when a = b and c is of sign opposite to that of a.

2. The total number of local maxima and local minima of the function $f(x) = \begin{cases} (2+x)^3, & -3 < x \le -1 \\ x^{2/3}, & -1 < x < 2 \end{cases}$ is

(A) 0
(B) 1
(C) 2
(D) 3

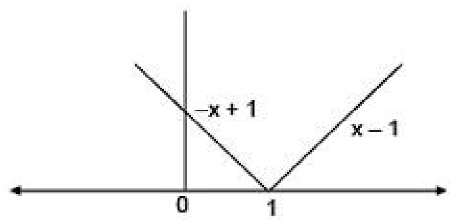
Sol. (C)
Local maximum at x = – 1
and local minimum at x = 0
Hence total number of local maxima and local minima is 2.



3. Let $g(x) = \frac{(x-1)^n}{\log\cos^m(x-1)}$; $0 < x < 2$, m and n are integers, $m \ne 0$, $n > 0$, and let p be the left hand derivative of $|x - 1|$ at $x = 1$. If $\lim_{x\to 1^+} g(x) = p$, then

(A) n = 1, m = 1
(B) n = 1, m = – 1
(C) n = 2, m = 2
(D) n > 2, m = n

Sol. (C)
From graph, p = – 1



$\Rightarrow \lim_{x\to 1^+} g(x) = -1$
$\Rightarrow \lim_{h\to 0} g(1+h) = -1$
$\Rightarrow \lim_{h\to 0}\left(\frac{h^n}{\log\cos^m h}\right) = -1$
$\Rightarrow \lim_{h\to 0}\frac{n\cdot h^{n-1}}{m\cdot(-\tan h)} = -\left(\frac{n}{m}\right)\lim_{h\to 0}\left(\frac{h^{n-1}}{\tan h}\right) = -1$, which holds if n = m = 2.

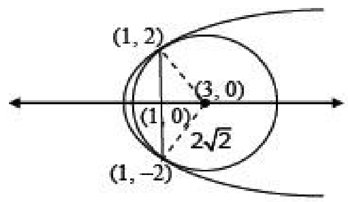
4. If $0 < x < 1$, then $\sqrt{1+x^2}\left[\{x\cos(\cot^{-1}x) + \sin(\cot^{-1}x)\}^2 - 1\right]^{1/2}$ is equal to

(A) $\frac{x}{\sqrt{1+x^2}}$
(B) x
(C) $x\sqrt{1+x^2}$
(D) $\sqrt{1+x^2}$

Sol. (C)
$\sqrt{1+x^2}\left[(x\cos\cot^{-1}x + \sin\cot^{-1}x)^2 - 1\right]^{1/2}$
$= \sqrt{1+x^2}\left[\left(x\cos\cos^{-1}\frac{x}{\sqrt{1+x^2}} + \sin\sin^{-1}\frac{1}{\sqrt{1+x^2}}\right)^2 - 1\right]^{1/2}$
$= \sqrt{1+x^2}\left[\left(\frac{x^2}{\sqrt{1+x^2}} + \frac{1}{\sqrt{1+x^2}}\right)^2 - 1\right]^{1/2}$
$= \sqrt{1+x^2}(x^2+1-1)^{1/2} = x\sqrt{1+x^2}$.

5. Consider the two curves $C_1 : y^2 = 4x$, $C_2 : x^2 + y^2 - 6x + 1 = 0$. Then,
(A) C_1 and C_2 touch each other only at one point
(B) C_1 and C_2 touch each other exactly at two points
(C) C_1 and C_2 intersect (but do not touch) at exactly two points
(D) C_1 and C_2 neither intersect nor touch each other

Sol. **(B)**
The circle and the parabola touch each other at x = 1 i.e. at the points (1, 2) and (1, – 2) as shown in the figure.



6. The edges of a parallelopiped are of unit length and are parallel to non-coplanar unit vectors $\hat{a}, \hat{b}, \hat{c}$ such that $\hat{a}\cdot\hat{b} = \hat{b}\cdot\hat{c} = \hat{c}\cdot\hat{a} = 1/2$. Then the volume of the parallelopiped is

(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{1}{2\sqrt{2}}$

(C) $\frac{\sqrt{3}}{2}$ (D) $\frac{1}{\sqrt{3}}$

Sol. **(A)**

$$\text{Volume} = \left|\hat{a}\cdot(\hat{b}\times\hat{c})\right| = \sqrt{\begin{vmatrix} \hat{a}\cdot\hat{a} & \hat{a}\cdot\hat{b} & \hat{a}\cdot\hat{c} \\ \hat{b}\cdot\hat{a} & \hat{b}\cdot\hat{b} & \hat{b}\cdot\hat{c} \\ \hat{c}\cdot\hat{a} & \hat{c}\cdot\hat{b} & \hat{c}\cdot\hat{c} \end{vmatrix}}$$

$$= \sqrt{\begin{vmatrix} 1 & 1/2 & 1/2 \\ 1/2 & 1 & 1/2 \\ 1/2 & 1/2 & 1 \end{vmatrix}} = \frac{1}{\sqrt{2}}.$$

SECTION – II

Multiple Correct Answers Type

This section contains 4 multiple correct answer(s) type questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONE OR MORE** is/are correct.

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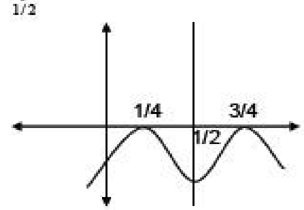
7. Let f (x) be a non-constant twice differentiable function defined on (– ∞, ∞) such that f (x) = f (1 – x) and $f'\left(\frac{1}{4}\right) = 0$. Then

(A) f″ (x) vanishes at least twice on [0, 1] (B) $f'\left(\frac{1}{2}\right) = 0$

(C) $\int_{-1/2}^{1/2} f\left(x+\frac{1}{2}\right)\sin x\,dx = 0$ (D) $\int_0^{1/2} f(t)e^{\sin \pi t}dt = \int_{1/2}^{1} f(1-t)e^{\sin \pi t}dt$

Sol. **(A, B, C, D)**
f (x) = f (1 – x)
Put x = 1/2 + x
$f\left(\frac{1}{2}+x\right) = f\left(\frac{1}{2}-x\right)$



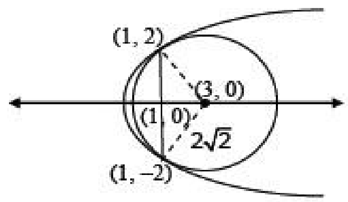
Hence f (x + 1/2) is an even function or f (x + 1/2) sin x is an odd function.
Also, f′ (x) = – f′ (1 – x) and for x = 1/2, we have $f'(1/2) = 0$.

Also, $\int_{1/2}^{1} f(1-t)e^{\sin \pi t}dt = -\int_{1/2}^{0} f(y)e^{\sin \pi y}dy$ (obtained by putting, 1 – t = y).

Since f′(1/4) = 0, f′ (3/4) = 0. Also f′ (1/2) = 0
⇒ f″ (x) = 0 atleast twice in [0, 1] (Rolle's Theorem)

5. Consider the two curves $C_1 : y^2 = 4x$, $C_2 : x^2 + y^2 - 6x + 1 = 0$. Then,
(A) C_1 and C_2 touch each other only at one point
(B) C_1 and C_2 touch each other exactly at two points
(C) C_1 and C_2 intersect (but do not touch) at exactly two points
(D) C_1 and C_2 neither intersect nor touch each other

Sol. **(B)**
The circle and the parabola touch each other at x = 1 i.e. at the points (1, 2) and (1, – 2) as shown in the figure.



6. The edges of a parallelopiped are of unit length and are parallel to non-coplanar unit vectors $\hat{a}, \hat{b}, \hat{c}$ such that $\hat{a} \cdot \hat{b} = \hat{b} \cdot \hat{c} = \hat{c} \cdot \hat{a} = 1/2$. Then the volume of the parallelopiped is

(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{1}{2\sqrt{2}}$

(C) $\frac{\sqrt{3}}{2}$ (D) $\frac{1}{\sqrt{3}}$

Sol. **(A)**

$$\text{Volume} = \left|\hat{a} \cdot (\hat{b} \times \hat{c})\right| = \sqrt{\begin{vmatrix} \hat{a}\cdot\hat{a} & \hat{a}\cdot\hat{b} & \hat{a}\cdot\hat{c} \\ \hat{b}\cdot\hat{a} & \hat{b}\cdot\hat{b} & \hat{b}\cdot\hat{c} \\ \hat{c}\cdot\hat{a} & \hat{c}\cdot\hat{b} & \hat{c}\cdot\hat{c} \end{vmatrix}}$$

$$= \sqrt{\begin{vmatrix} 1 & 1/2 & 1/2 \\ 1/2 & 1 & 1/2 \\ 1/2 & 1/2 & 1 \end{vmatrix}} = \frac{1}{\sqrt{2}}.$$

SECTION – II

Multiple Correct Answers Type

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---

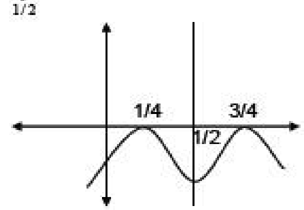
7. Let f (x) be a non-constant twice differentiable function defined on (– ∞, ∞) such that f (x) = f (1 – x) and $f'\left(\frac{1}{4}\right) = 0$.
Then

(A) f'' (x) vanishes at least twice on [0, 1] (B) $f'\left(\frac{1}{2}\right) = 0$

(C) $\int_{-1/2}^{1/2} f\left(x + \frac{1}{2}\right)\sin x\, dx = 0$ (D) $\int_{0}^{1/2} f(t)e^{\sin \pi t}dt = \int_{1/2}^{1} f(1-t)e^{\sin \pi t}dt$

Sol. **(A, B, C, D)**
f (x) = f (1 – x)
Put x = 1/2 + x

$$f\left(\frac{1}{2} + x\right) = f\left(\frac{1}{2} - x\right)$$



Hence f (x + 1/2) is an even function or f (x + 1/2) sin x is an odd function.
Also, f' (x) = – f' (1 – x) and for x = 1/2, we have $f'(1/2) = 0$.

Also, $\int_{1/2}^{1} f(1-t)e^{\sin \pi t}dt = -\int_{1/2}^{0} f(y)e^{\sin \pi y}dy$ (obtained by putting, 1 – t = y).

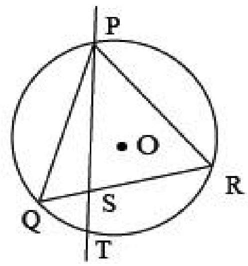
Since f'(1/4) = 0, f' (3/4) = 0. Also f' (1/2) = 0
⇒ f'' (x) = 0 atleast twice in [0, 1] (Rolle's Theorem)

8. A straight line through the vertex P of a triangle PQR intersects the side QR at the point S and the circumcircle of the triangle PQR at the point T. If S is not the centre of the circumcircle, then

(A) $\frac{1}{PS}+\frac{1}{ST}<\frac{2}{\sqrt{QS\times SR}}$ (B) $\frac{1}{PS}+\frac{1}{ST}>\frac{2}{\sqrt{QS\times SR}}$

(C) $\frac{1}{PS}+\frac{1}{ST}<\frac{4}{QR}$ (D) $\frac{1}{PS}+\frac{1}{ST}>\frac{4}{QR}$

Sol. **(B, D)**



$PS \times ST = QS \times SR$

$$\frac{\frac{1}{PS}+\frac{1}{ST}}{2} > \sqrt{\frac{1}{PS}\times\frac{1}{ST}}$$

$$\Rightarrow \frac{1}{PS}+\frac{1}{ST} > \frac{2}{\sqrt{QS\times SR}}$$

$$\frac{QS+SR}{2} > \sqrt{QS\times SR}$$

$$\frac{QR}{2} > \sqrt{QS\times SR} \quad \Rightarrow \quad \frac{1}{\sqrt{QS\times SR}} > \frac{2}{QR}$$

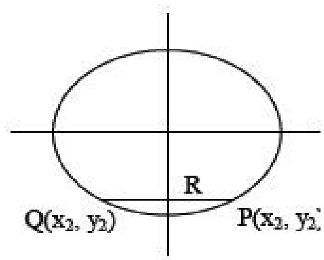
$$\Rightarrow \frac{1}{PS}+\frac{1}{ST} > \frac{4}{QR}.$$

9. Let P (x_1, y_1) and Q (x_2, y_2), $y_1 < 0$, $y_2 < 0$, be the end points of the latus rectum of the ellipse $x^2 + 4y^2 = 4$. The equations of parabolas with latus rectum PQ are

(A) $x^2+2\sqrt{3}\,y=3+\sqrt{3}$ (B) $x^2-2\sqrt{3}\,y=3+\sqrt{3}$

(C) $x^2+2\sqrt{3}\,y=3-\sqrt{3}$ (D) $x^2-2\sqrt{3}\,y=3-\sqrt{3}$

Sol. **(B, C)**



$$\frac{x^2}{4}+\frac{y^2}{1}=1$$

$b^2 = a^2 (1 - e^2)$

$$\Rightarrow e = \frac{\sqrt{3}}{2}$$

$\Rightarrow P\left(\sqrt{3}, -\frac{1}{2}\right)$ and $Q\left(-\sqrt{3}, -\frac{1}{2}\right)$ (given y_1 and y_2 less than 0).

Co-ordinates of mid-point of PQ are

$R \equiv \left(0, -\frac{1}{2}\right)$.

PQ = $2\sqrt{3}$ = length of latus rectum.

$\Rightarrow$ two parabola are possible whose vertices are $\left(0, -\frac{\sqrt{3}}{2}-\frac{1}{2}\right)$ and $\left(0, \frac{\sqrt{3}}{2}-\frac{1}{2}\right)$.

Hence the equations of the parabolas are $x^2 - 2\sqrt{3}y = 3+\sqrt{3}$

and $x^2+2\sqrt{3}y = 3-\sqrt{3}$.

10. Let $S_n = \sum_{k=1}^{n}\frac{n}{n^2+kn+k^2}$ and $T_n = \sum_{k=0}^{n-1}\frac{n}{n^2+kn+k^2}$ for n = 1, 2, 3, …. Then,

(A) $S_n < \frac{\pi}{3\sqrt{3}}$ (B) $S_n > \frac{\pi}{3\sqrt{3}}$

(C) $T_n < \frac{\pi}{3\sqrt{3}}$ (D) $T_n > \frac{\pi}{3\sqrt{3}}$

Sol. **(A, D)**

$S_n < \lim_{n\to\infty} S_n = \lim_{n\to\infty}\sum_{k=1}^{n}\frac{1}{n}\cdot\frac{1}{1+k/n+(k/n)^2}$

$= \int_0^1 \frac{dx}{1+x+x^2} = \frac{\pi}{3\sqrt{3}}$

Now, $T_n > \frac{\pi}{3\sqrt{3}}$ as $h\sum_{k=0}^{n-1} f(kh) > \int_0^1 f(x)dx > h\sum_{k=1}^{n} f(kh)$

SECTION – III

Reasoning Type

This section contains 4 reasoning type questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

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11. Consider the system of equations $ax + by = 0$, $cx + dy = 0$, where $a, b, c, d \in \{0, 1\}$.

STATEMENT – 1: The probability that the system of equations has a unique solution is 3/8.

and

STATEMENT – 2: The probability that the system of equations has a solution is 1.

(A) Statement-1 is True, Statement -2 is True; Statement-2 is a correct explanation for Statement-1

(B) Statement -1 is True, Statement -2 is True; Statement-2 is **NOT** a correct explanation for Statement-1

(C) Statement -1 is True, Statement -2 is False

(D) Statement -1 is False, Statement -2 is True

Sol. **(B)**

For unique solution $\begin{vmatrix} a & b \\ c & d \end{vmatrix} \neq 0$ where $a, b, c, d \in \{0, 1\}$

Total cases = 16.

Favorable cases = 6 (Either ad = 1, bc = 0 or ad = 0, bc = 1).

Probability that system of equations has unique solution is $\frac{6}{16} = \frac{3}{8}$ and system of equations has either unique solution or infinite solutions so that probability for system to have a solution is 1.

12. Consider the system of equations

$x - 2y + 3z = -1$

$-x + y - 2z = k$

$x - 3y + 4z = 1$

STATEMENT -1 : The system of equations has no solution for $k \neq 3$.

and

STATEMENT -2 : The determinant $\begin{vmatrix} 1 & 3 & -1 \\ -1 & -2 & k \\ 1 & 4 & 1 \end{vmatrix} \neq 0$, for $k \neq 3$.

(A) Statement-1 is True, Statement -2 is True; Statement-2 is a correct explanation for Statement-1

(B) Statement -1 is True, Statement -2 is True; Statement-2 is **NOT** a correct explanation for Statement-1

(C) Statement -1 is True, Statement -2 is False

(D) Statement -1 is False, Statement -2 is True

Sol. **(A)**

$$D = \begin{vmatrix} 1 & -2 & 3 \\ -1 & 1 & -2 \\ 1 & -3 & 4 \end{vmatrix} = 0$$

$$\text{and } D_1 = \begin{vmatrix} -1 & -2 & 3 \\ k & 1 & -2 \\ 1 & -3 & 4 \end{vmatrix} = (3-k) = 0 \text{ if } k = 3$$

$$D_2 = \begin{vmatrix} 1 & -1 & 3 \\ -1 & k & -2 \\ 1 & 1 & 4 \end{vmatrix} = (k-3) = 0\text{, if } k = 3$$

$$D_3 = \begin{vmatrix} 1 & -2 & -1 \\ -1 & 1 & k \\ 1 & -3 & 1 \end{vmatrix} = (k-3) = 0\text{, if } k = 3$$

$\Rightarrow$ system of equations has no solution for $k \neq 3$.

13. Let f and g be real valued functions defined on interval (–1, 1) such that $g''(x)$ is continuous, $g(0) \neq 0$, $g'(0) = 0$, $g''(0) \neq 0$, and $f(x) = g(x) \sin x$.

STATEMENT -1 : $\lim_{x\to 0}[g(x)\cot x - g(0)\operatorname{cosec} x] = f''(0)$.

and

STATEMENT -2 : $f'(0) = g(0)$.

(A) Statement-1 is True, Statement -2 is True; Statement-2 is a correct explanation for Statement-1
(B) Statement -1 is True, Statement -2 is True; Statement-2 is **NOT** a correct explanation for Statement-1
(C) Statement -1 is True, Statement -2 is False
(D) Statement -1 is False, Statement -2 is True

Sol. **(B)**

$f'(x) = g(x)\cos x + \sin x . g'(x)$

$\Rightarrow f'(0) = g(0)$

$f''(x) = 2g'(x)\cos x - g(x)\sin x + \sin x\, g''(x)$

$\Rightarrow f''(0) = 2g'(0) = 0$

But $\lim_{x\to 0}\left[g(x)\cot x - g(0)\operatorname{cosec} x\right] = \lim_{x\to 0}\frac{g(x)\cos x - g(0)}{\sin x} = \lim_{x\to 0}\frac{g'(x)\cos x - g(x)\sin x}{\cos x} = g'(0) = 0 = f''(0)$.

14. Consider three planes

$P_1 : x - y + z = 1$

$P_2 : x + y - z = -1$

$P_3 : x - 3y + 3z = 2$.

Let L_1, L_2, L_3 be the lines of intersection of the planes P_2 and P_3, P_3 and P_1, and P_1 and P_2, respectively.

STATEMENT -1 : At least two of the lines L_1, L_2 and L_3 are non-parallel.

and

STATEMENT -2 : The three planes do not have a common point.

(A) Statement-1 is True, Statement -2 is True; Statement-2 is a correct explanation for Statement-1
(B) Statement -1 is True, Statement -2 is True; Statement-2 is **NOT** a correct explanation for Statement-1
(C) Statement -1 is True, Statement -2 is False
(D) Statement -1 is False, Statement -2 is True

Sol. **(D)**

The direction cosines of each of the lines L_1, L_2, L_3 are proportional to (0, 1, 1).

SECTION – IV

Linked Comprehension Type

This section contains 3 paragraphs. Based upon each paragraph, 3 multiple choice questions have to be answered. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

Paragraph for Question Nos. 15 to 17

Consider the functions defined implicitly by the equation $y^3 - 3y + x = 0$ on various intervals in the real line.
If $x \in (-\infty, -2) \cup (2, \infty)$, the equation implicitly defines a unique real valued differentiable function $y = f(x)$.
If $x \in (-2, 2)$, the equation implicitly defines a unique real valued differentiable function $y = g(x)$ satisfying $g(0) = 0$.

15. If $f\left(-10\sqrt{2}\right) = 2\sqrt{2}$, then $f''\left(-10\sqrt{2}\right) =$

(A) $\frac{4\sqrt{2}}{7^3 3^2}$ (B) $-\frac{4\sqrt{2}}{7^3 3^2}$

(C) $\frac{4\sqrt{2}}{7^3 3}$ (D) $-\frac{4\sqrt{2}}{7^3 3}$

Sol. **(B)**
Differentiating the given equation, we get
$3y^2y' - 3y' + 1 = 0$
$\Rightarrow y'\left(-10\sqrt{2}\right) = -\frac{1}{21}$
Differentiation again we get $6yy'^2 + 3y^2y'' - 3y'' = 0$
$\Rightarrow f''\left(-10\sqrt{2}\right) = -\frac{6.2\sqrt{2}}{(21)^4} = -\frac{4\sqrt{2}}{7^3 3^2}$.

16. The area of the region bounded by the curves $y = f(x)$, the x-axis, and the lines $x = a$ and $x = b$, where $-\infty < a < b < -2$, is

(A) $\int_a^b \frac{x}{3\left((f(x))^2 - 1\right)}dx + bf(b) - af(a)$ (B) $-\int_a^b \frac{x}{3\left((f(x))^2 - 1\right)}dx + bf(b) - af(a)$

(C) $\int_a^b \frac{x}{3\left((f(x))^2 - 1\right)}dx - bf(b) + af(a)$ (D) $-\int_a^b \frac{x}{3\left((f(x))^2 - 1\right)}dx - bf(b) + af(a)$

Sol. **(A)**

The required area = $\int_a^b f(x)dx = xf(x)\Big|_a^b - \int_a^b xf'(x)dx$

$$= bf(b) - af(a) + \int_a^b \frac{x}{3\left[(f(x)^2 - 1)\right]}dx.$$

17. $\int_{-1}^{1} g'(x)dx =$

(A) $2g(-1)$ (B) 0
(C) $-2g(1)$ (D) $2g(1)$

Sol. **(D)**

We have $y' = \frac{1}{3\left(1 - \left(f(x)^2\right)\right)}$ which is even

Hence $\int_{-1}^{1} g'(x) = g(1) - g(-1) = 2g(1)$.

Paragraph for Question Nos. 18 to 20

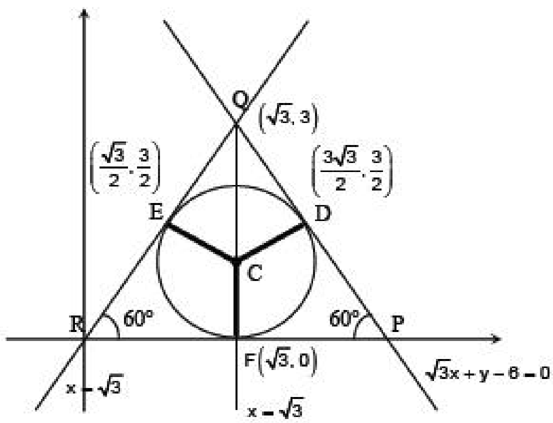
A circle C of radius 1 is inscribed in an equilateral triangle PQR. The points of contact of C with the sides PQ, QR, RP are D, E, F, respectively. The line PQ is given by the equation $\sqrt{3}x + y - 6 = 0$ and the point D is $\left(\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$. Further, it is given that the origin and the centre of C are on the same side of the line PQ.

18. The equation of circle C is

(A) $\left(x - 2\sqrt{3}\right)^2 + (y-1)^2 = 1$ (B) $\left(x - 2\sqrt{3}\right)^2 + \left(y + \frac{1}{2}\right)^2 = 1$

(C) $\left(x - \sqrt{3}\right)^2 + (y+1)^2 = 1$ (D) $\left(x - \sqrt{3}\right)^2 + (y-1)^2 = 1$

Sol. **(D)**



Equation of CD is $\dfrac{x - \frac{3\sqrt{3}}{2}}{\frac{\sqrt{3}}{2}} = \dfrac{y - \frac{3}{2}}{\frac{1}{2}} = -1$

$\Rightarrow C \equiv \left(\sqrt{3}, 1\right)$

Equation of the circle is $\left(x - \sqrt{3}\right)^2 + (y-1)^2 = 1$.

19. Points E and F are given by

(A) $\left(\frac{\sqrt{3}}{2}, \frac{3}{2}\right), \left(\sqrt{3}, 0\right)$ (B) $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right), \left(\sqrt{3}, 0\right)$

(C) $\left(\frac{\sqrt{3}}{2}, \frac{3}{2}\right), \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ (D) $\left(\frac{3}{2}, \frac{\sqrt{3}}{2}\right), \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

Sol. **(A)**

Since the radius of the circle is 1 and C $\left(\sqrt{3}, 1\right)$, coordinates of $F \equiv \left(\sqrt{3}, 0\right)$

Equation of CE is $\dfrac{x - \sqrt{3}}{-\frac{\sqrt{3}}{2}} = \dfrac{y-1}{\frac{1}{2}} = 1$

$\Rightarrow E \equiv \left(\frac{\sqrt{3}}{2}, \frac{3}{2}\right)$.

20. Equation of the sides QR, RP are

(A) $y = \frac{2}{\sqrt{3}}x + 1, y = -\frac{2}{\sqrt{3}}x - 1$

(B) $y = \frac{1}{\sqrt{3}}x,\ y = 0$

(C) $y = \frac{\sqrt{3}}{2}x + 1, y = -\frac{\sqrt{3}}{2}x - 1$

(D) $y = \sqrt{3}x,\ y = 0$

Sol. **(D)**

Equation of QR is $y - 3 = \sqrt{3}\left(x - \sqrt{3}\right)$

$\Rightarrow\ y = \sqrt{3x}$

Equation of RP is $y = 0$.

Paragraph for Question Nos. 21 to 23

Let A, B, C be three sets of complex numbers as defined below

$A = \{z : \text{Im} z \ge 1\}$

$B = \{z : |z - 2 - i| = 3\}$

$C = \{z : \text{Re}((1 - i)z) = \sqrt{2}\}$.

21. The number of elements in the set $A \cap B \cap C$ is

(A) 0

(B) 1

(C) 2

(D) ∞

Sol. **(B)**

A = Set of points on and above the line y = 1 in the Argand plane.

B = Set of points on the circle $(x - 2)^2 + (y - 1)^2 = 3^2$

$C = \text{Re}(1 - i)z = \text{Re}((1 - i)(x + iy))$

$\Rightarrow x + y = \sqrt{2}$

Hence $(A \cap B \cap C)$ = has only one point of intersection.

22. Let z be any point in $A \cap B \cap C$. Then, $|z + 1 - i|^2 + |z - 5 - i|^2$ lies between

(A) 25 and 29

(B) 30 and 34

(C) 35 and 39

(D) 40 and 44

Sol. **(C)**

The points (– 1, 1) and (5, 1) are the extremities of a diameter of the given circle .

Hence $|z + 1 - i|^2 + |z - 5 - i|^2 = 36$.

23. Let z be any point in $A \cap B \cap C$ and let w be any point satisfying $|w - 2 - i| < 3$. Then, $|z| - |w| + 3$ lies between

(A) –6 and 3

(B) –3 and 6

(C) –6 and 6

(D) –3 and 9

Sol. **(D)**

$\left||z| - |w|\right| < |z - w|$

and $|z - w|$ = Distance between z and w

z is fixed. Hence distance between z and w would be maximum for diametrically opposite points.

$\Rightarrow |z - w| < 6$

$\Rightarrow -6 < |z| - |w| < 6$

$\Rightarrow -3 < |z| - |w| + 3 < 9$.