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Paper - I
“2011”
(Math)

MATHEMATICS PAPER - I
PART-III- MATHEMATICS
SECTION-1 (TOTAL MARKS: 21)
(SINGLE CORRECT ANSWER TYPE)

This section contains 7 multiple choice questions. Each question has four choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

47. Let $P = \{\theta : \sin \theta - \cos \theta = \sqrt{2} \cos \theta\}$ and $Q = \{\theta : \sin \theta + \cos \theta = \sqrt{2} \sin \theta\}$ be two sets. Then
 (a) $P \subset Q$ and $Q - P \neq \emptyset$ (b) $Q \subset P$ (c) $P \subset Q$ (d) $P = Q$

Ans: (d) $P : \sin \theta - \cos \theta = \sqrt{2} \cos \theta \Rightarrow \tan \theta = 1 + \sqrt{2}$
 $Q : \sin \theta + \cos \theta = \sqrt{2} \sin \theta \Rightarrow \tan \theta = \frac{1}{\sqrt{2} - 1} = \sqrt{2} + 1$
 $\therefore P = Q$

48. Let the straight line $x = b$ divide the area enclosed by $y = (1 - x)^2$, $y = 0$ and $x = 0$ into two parts $R_1 (0 \leq x \leq b)$ and $R_2 (0 \leq x \leq 1)$ such that $R_1 - R_2 = \frac{1}{4}$. Then b equals
 (a) $\frac{3}{4}$ (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{1}{4}$

Ans: (b) $R_1 - R_2 = \frac{1}{4} \Rightarrow \int_0^b (1 - x^2) dx - \int_b^1 (1 - x)^2 dx = \frac{1}{4} \Rightarrow \frac{1}{3} - \frac{2(1 - b)^3}{3} = \frac{1}{4}$
 $\Rightarrow (1 - b)^3 = \frac{1}{8} \Rightarrow b = \frac{1}{2}$

49. Let α and β be the roots of $x^2 - 6x - 2 = 0$, with $\alpha > \beta$. If $a_n = \alpha^n - \beta^n$ for $n \geq 1$, then the value of $\frac{a_{10} - 2a_8}{2a_9}$ is
 (a) 1 (b) 2 (c) 3 (d) 4

Ans: (c) Since $\alpha^2 - 6\alpha - 2 = 0 \Rightarrow \alpha^2 - 2 = 6\alpha$
 and $\beta^2 - 6\beta - 2 = 0 \Rightarrow \beta^2 - 2 = 6\beta$
 $\frac{a_{10} - 2a_8}{2a_9} = \frac{(\alpha^{10} - \beta^{10}) - 2(\alpha^8 - \beta^8)}{2(\alpha^9 - \beta^9)} = \frac{\alpha^8(\alpha^2 - 2) - \beta^8(\beta^2 - 2)}{2(\alpha^9 - \beta^9)} = \frac{\alpha^8 \cdot 6\alpha - \beta^8 \cdot 6\beta}{2(\alpha^9 - \beta^9)} = 3$

50. A straight line L through the point (3, -2) is inclined at an angle 60° to the line $\sqrt{3}x + y = 1$. If L also intersects the x-axis, then the equation of L is

- (a) $y + \sqrt{3}x + 2 - 3\sqrt{3} = 0$ (b) $y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$
 (c) $\sqrt{3}y - x + 3 + 2\sqrt{3} = 0$ (d) $\sqrt{3}y + x - 3 + 2\sqrt{3} = 0$

Ans: (b) If required line has slope = m

$$\Rightarrow \left| \frac{m + \sqrt{3}}{1 - \sqrt{3}m} \right| = \tan 60^\circ \Rightarrow m = 0, \sqrt{3}$$

Since, the line intersects the x-axis

$$\Rightarrow \text{equation is, } (y + 2) = \sqrt{3}(x - 3) \Rightarrow y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$$

51. Let (x_0, y_0) be the solution of the following equations

$$(2x)^{\ln 2} = (3y)^{\ln 3}$$

$$3^{\ln x} = 2^{\ln y}$$

Then x_0 is

- (a) $1/6$ (b) $1/3$ (c) $1/2$ (d) 6

Ans: (c) Since $(\ln 2)(\ln 2 + \ln x) = (\ln 3)(\ln 3 + \ln y)$ and $\ln x \cdot \ln 3 - \ln y \cdot \ln 2 = 0$

$$\Rightarrow (\ln 2) \cdot (\ln x) - \ln y \cdot \ln 3 = (\ln 3)^2 - (\ln 2)^2 \quad \dots\dots(i)$$

$$\text{and } \ln x \cdot \ln 3 - \ln y \cdot \ln 2 = 0 \quad \dots\dots(ii)$$

Using (i) $\times \ln 2 -$ (ii) $\times \ln 3$, we get

$$\ln x \cdot \{(\ln 2)^2 - (\ln 3)^2\} = (\ln 2) \{(\ln 3)^2 - (\ln 2)^2\}$$

$$\Rightarrow \ln x = -\ln 2 \Rightarrow x = 1/2$$

52. The value of $\int_{\sqrt{\ln 2}}^{\sqrt{\ln 3}} \frac{x \sin x^2}{\sin x^2 + \sin(\ln 6 - x^2)} dx$ is

- (a) $(1/4)\ln(3/2)$ (b) $(1/2)\ln(3/2)$ (c) $\ln(3/2)$ (d) $(1/6)\ln(3/2)$

Ans: (a) $I = \int_{\sqrt{\ln 2}}^{\sqrt{\ln 3}} \frac{x \sin x^2}{\sin x^2 + \sin(\ln 6 - x^2)} dx$

$$x^2 = t \Rightarrow 2x dx = dt$$

$$\Rightarrow I = \frac{1}{2} \int_{\ln 2}^{\ln 3} \frac{\sin t}{\sin t + \sin(\ln 6 - t)} dt$$

$$\text{Using } \int_a^b f(x) dx = \frac{1}{2} \int_a^b (f(x) + f(a+b-x)) dx$$

$$\Rightarrow I = \frac{1}{4} \int_{\ln 2}^{\ln 3} \left(\frac{\sin t}{\sin t + \sin(\ln 6 - t)} + \frac{\sin(\ln 6 - t)}{\sin(\ln 6 - t) + \sin t} \right) dt \quad [\text{since } \ln 3 + \ln 2 = \ln 6]$$

$$\Rightarrow I = \frac{1}{4} \int_{\ln 2}^{\ln 3} dt = \frac{1}{4} [t]_{\ln 2}^{\ln 3} = \frac{1}{4} \ln \frac{3}{2}$$

53. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} - \hat{k}$ be three vectors. A vector \vec{v} in the plane of \vec{a} and \vec{b} , whose projection on \vec{c} is $1/\sqrt{3}$ is given by
 (a) $\hat{i} - 3\hat{j} + 3\hat{k}$ (b) $-3\hat{i} - 3\hat{j} - \hat{k}$ (c) $3\hat{i} - \hat{j} + 3\hat{k}$ (d) $\hat{i} + 3\hat{j} - 3\hat{k}$

Ans: (c) $\vec{v} = \lambda(\hat{i} + \hat{j} + \hat{k}) + \mu(\hat{i} - \hat{j} + \hat{k})$
 $\frac{\vec{v} \cdot \vec{c}}{|\vec{c}|} = \frac{1}{\sqrt{3}} \Rightarrow (\lambda + \mu) - (\lambda - \mu) - (\lambda + \mu) = 1 \Rightarrow \mu = \lambda + 1$
 $\vec{v} = \hat{i}(2\lambda + 1) - \hat{j} + \hat{k}(2\lambda + 1)$
 Only option (c) satisfies this one.

SECTION-II (TOTAL MARKS: 16)
(Multiple Correct Choice Type)

This section contains 4 multiple choice questions. Each question has four choices (a), (b), (c) and (d) out of which **ONE OR MORE** may be correct.

54. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function such that $f(x+y) = f(x) + f(y)$, $\forall x, y \in \mathbb{R}$.
 If $f(x)$ is differentiable at $x = 0$, then
 (a) $f(x)$ is differentiable only in a finite interval containing zero.
 (b) $f(x)$ is continuous $\forall x \in \mathbb{R}$
 (c) $f'(x)$ is constant $\forall x \in \mathbb{R}$
 (d) $f(x)$ is differentiable except at finitely many points

Ans: (b) (c) $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{f(x) + f(h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{f(h) - f(0)}{h}$
 Put $x = y = 0$, $f(0) = 0$
 $f'(x) = f'(0)$
 $f(x) = f'(0)x + c$
 $f(x) = kx + c$, $c = 0$ as $f(0) = 0$
 \therefore Function is continuous. $f'(x)$ is constant.

55. Let the eccentricity of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be reciprocal to that of the ellipse $x^2 + 4y^2 = 4$.
 If the hyperbola passes through a focus of the ellipse, then
 (a) the equation of the hyperbola is $\frac{x^2}{3} - \frac{y^2}{2} = 1$
 (b) a focus of the hyperbola is $(2, 0)$
 (c) the eccentricity of the hyperbola is $\sqrt{\frac{5}{3}}$
 (d) the equation of the hyperbola is $x^2 - 3y^2 = 3$

Ans: (b) (d) $\frac{x^2}{4} + \frac{y^2}{1} = 1$, $a^2 = 4, b^2 = 1, e^2 = 1 - (b^2/a^2) \Rightarrow e = \sqrt{3}/2$
 eccentricity of hyperbola = $2/\sqrt{3}$
 Focus of ellipse = $(\pm ae, 0) = (\pm\sqrt{3}, 0)$
 $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ passes through focus $(\pm\sqrt{3}, 0)$
 $\therefore a^2 = 3$ $\therefore b^2 = a^2(e^2 - 1) = 1$
 \therefore Equation of hyperbola: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \Rightarrow \frac{x^2}{3} - \frac{y^2}{1} = 1$
 $x^2 - 3y^2 = 3$
 Focus of hyperbola = $(\pm ae, 0) = (\pm 2, 0)$

56. Let M and N be two 3×3 non-singular skew-symmetric matrices such that $MN = NM$. If P^T denotes the transpose of P , then $M^2 N^2 (M^T N)^{-1} (MN^{-1})^T$ is equal to
 (a) M^2 (b) $-N^2$ (c) $-M^2$ (d) MN

Ans: (c) Since $M^T = -M$, $N^T = -N$

$$M^2 N^2 (M^T N)^{-1} (MN^{-1})^T$$

$$= M^2 N^2 (-MN)^{-1} (MN^{-1})^T = M^2 N^2 (N^{-1} \cdot (-M)^{-1}) \cdot (N^T)^{-1} \cdot (-M)$$

$$= M^2 N (-M)^{-1} \cdot (-N)^{-1} \cdot (-M) = M(MN)(MN)^{-1} (-M) = -M^2$$

57. The vector (s) which is/are coplanar with vectors $\hat{i} + \hat{j} + 2\hat{k}$ and $\hat{i} + 2\hat{j} + \hat{k}$, and perpendicular to the vector $\hat{i} + \hat{j} + \hat{k}$ is /are
 (a) $\hat{j} - \hat{k}$ (b) $-\hat{i} + \hat{j}$ (c) $\hat{i} - \hat{j}$ (d) $-\hat{j} + \hat{k}$

Ans: (a) (d) Let $\vec{a} = \lambda(\hat{i} + \hat{j} + 2\hat{k}) + \mu(\hat{i} + 2\hat{j} + \hat{k})$

$$= (\lambda + \mu)\hat{i} + (\lambda + 2\mu)\hat{j} + (2\lambda + \mu)\hat{k}$$

$$\vec{a} \cdot (\hat{i} + \hat{j} + \hat{k}) = 0 \quad \therefore \lambda = -\mu \quad \therefore \vec{a} = \lambda(-\hat{j} + \hat{k})$$

Taking $\lambda = 1$, $\vec{a} = -\hat{j} + \hat{k}$
 $\lambda = -1$, $\vec{a} = \hat{j} - \hat{k}$

SECTION-III (TOTAL MARKS: 15)
(Paragraph Type)

This section contains **2 paragraphs**. Based upon one of the paragraphs 2 multiple choice questions and based on other paragraph 3 multiple choice questions have to be answered. Each of these has four choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

Paragraph for question Nos 58 and 59

Let U_1 and U_2 be two urns such that U_1 contains 3 white and 2 red balls, and U_2 contains only 1 white ball. A fair coin is tossed. If head appears then 1 ball is drawn at random from U_1 and put into U_2 . However, if tail appears then 2 balls are drawn at random from U_1 and put into U_2 . Now 1 ball is drawn at random from U_2 .

58. The probability of the drawn ball from U_2 being white is
(a) 13/30 (b) 23/30 (c) 19/30 (d) 11/30

Ans : (b)

$$P = \frac{1}{2} \left[\frac{3}{5} \times 1 + \frac{2}{5} \times \frac{1}{2} \right] + \frac{1}{2} \left[\frac{{}^3C_1 \times {}^2C_1}{{}^5C_2} \times \frac{2}{3} + \frac{{}^3C_2 \times 1 + \frac{{}^2C_2 \times 1}{3}}{{}^5C_2} \right] = 23/30$$

For appearing head For appearing tail

59. Given that the drawn ball from U_2 is white, the probability that head appeared on the coin is
(a) 17/23 (b) 11/23 (c) 15/23 (d) 12/23

Ans : (d)

Using Bayes theorem

$$P = \frac{\frac{1}{2} \left[\frac{3}{5} \times 1 + \frac{2}{5} \times \frac{1}{2} \right]}{\frac{1}{2} \left[\frac{3}{5} \times 1 + \frac{2}{5} \times \frac{1}{2} \right] + \frac{1}{2} \left[\frac{{}^3C_1 \times {}^2C_1}{{}^5C_2} \times \frac{2}{3} + \frac{{}^3C_2 \times 1 + \frac{{}^2C_2 \times 1}{3}}{{}^5C_2} \right]} = 12/23$$

Paragraph for question Nos 60 and 62

Let a, b and c be three real numbers satisfying

$$\begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} 1 & 9 & 7 \\ 8 & 2 & 7 \\ 7 & 3 & 7 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} \quad \dots (E)$$

60. If the point P(a, b, c), with reference to (E), lies on the plane $2x + y + z = 1$, then the value of $7a + b + c$ is
(a) 0 (b) 12 (c) 7 (d) 6

Ans : (d) $D = \begin{vmatrix} 1 & 8 & 7 \\ 9 & 2 & 3 \\ 1 & 1 & 1 \end{vmatrix} = 0$

\therefore system has non trivial solution.

If $c = k$, $b = -6/7 k$, $a = -k/7$

$\therefore 2a + b + c = 1$

$$-(2k/7) - (6k/7) + k = 1 \Rightarrow k = -7$$

61. Let ω be a solution of $x^3 - 1 = 0$ with $\text{Im}(\omega) > 0$. If $a = 2$ with b and c satisfying (E), then the value of $\frac{3}{\omega^a} + \frac{1}{\omega^b} + \frac{3}{\omega^c}$ is equal to
 (a) -2 (b) 2 (c) 3 (d) -3

Ans: (a) $a = 2, k = -14$
 $c = -14, b = 12, a = 2$

$$\begin{aligned} \frac{3}{\omega^2} + \frac{1}{\omega^{12}} + \frac{3}{\omega^{-14}} &= \frac{3}{\omega^2} + \frac{1}{(\omega^3)^4} + \frac{3(\omega^3)^5}{\omega} \\ &= \frac{3}{\omega^2} + 1 + \frac{3}{\omega} = \frac{3 + \omega^2 + 3\omega}{\omega^2} = \frac{3(1 + \omega) + \omega^2}{\omega^2} = \frac{-3\omega^2 + \omega^2}{\omega^2} = -2 \end{aligned}$$

62. Let $b = 6$, with a and c satisfying (E). If α and β are the roots of the quadratic equation $ax^2 + bx + c = 0$, then $\sum_{n=0}^{\infty} \left(\frac{1}{\alpha} + \frac{1}{\beta} \right)^n$ is
 (a) 6 (b) 7 (c) 6/7 (d) ∞

Ans: (b) $b = 6, k = -7, c = -7, a = 1$
 Q.E. becomes $x^2 + 6x - 7 = 0$; $a + b = -6, ab = -7$

$$\sum_{n=0}^{\infty} \left(\frac{\alpha + \beta}{\alpha\beta} \right)^n = \sum_{n=0}^{\infty} \left(\frac{6}{-7} \right)^n = 1 + \left(\frac{6}{-7} \right) + \left(\frac{6}{-7} \right)^2 + \dots = \frac{1}{1 - \frac{6}{-7}} = 7$$

SECTION-IV (TOTAL MARKS: 28)
(INTEGER ANSWER TYPE)

This section contains 7 questions. The answer to each of the questions is a **single – digit integer**, ranging from 0 to 9. The bubble corresponding to the correct answer is to be darkened in the ORS.

63. Let $f: [1, \infty) \rightarrow [(2, \infty)$ be a differentiable function such that $f(1) = 2$, if

$$6 \int_1^x f(t) dt = 3xf(x) - x^3$$

for all $x \geq 1$, then the value of $f(2)$ is

Ans: 6 Differentiating the given relation,

$$6f(x) = 3f(x) + 3xf'(x) - 3x^2 \Rightarrow f'(x) - \frac{f(x)}{x} = x, \text{ which is in linear form with I.F.} = \frac{1}{x}$$

$$\text{Hence, } f(x) = x^2 + cx \quad \text{with } c = 1. \quad \therefore f(x) = x^2 + x. \quad \therefore f(2) = 6$$

64. If z is any complex number satisfying $|z - 3 - 2i| \leq 2$, then the minimum value of $|2z - 6 + 5i|$ is

Ans: 5 Expression $2 \left| z - 3 + \frac{5}{2}i \right|$ which is equivalent to double the distance between complex number z and $(3, -5/2)$ which lies on one the diameter $x = 3$ of circle given. Hence minimum value of expression = $2 \times \frac{5}{2} = 5$

65. Let a_1, a_2, \dots, a_{100} be an arithmetic progression with $a_1 = 3$ and $S_p = \sum_{i=1}^p a_i$, $1 \leq p \leq 100$. For any integer n with $1 \leq n \leq 20$, let $m = 5n$. If $\frac{S_m}{S_n}$ does not depend on n , then a_2 is

$$\text{Ans: 9} \quad \frac{S_m}{S_n} = \frac{a_1 + a_2 + \dots + a_{5n}}{a_1 + a_2 + \dots + a_n} = \frac{\frac{5n}{2} \{2.3 + (5n-1)d\}}{\frac{n}{2} \{2.3 + (n-1)d\}}$$

$$\lambda = \frac{5(6 + 5nd - d)}{(6 + nd - d)} \quad \left[\lambda = \frac{S_m}{S_n} \right]$$

$\therefore \lambda$ independent of n when $d = 6$

$$\therefore a_2 = a_1 + d = 9$$

66. Consider the parabola $y^2 = 8x$. Let Δ_1 be the area of the triangle formed by the end points of its latus rectum and the point $P\left(\frac{1}{2}, 2\right)$ on the parabola, and Δ_2 be the area of the triangle formed by drawing tangents at P and at the end points of the latus rectum. Then $\frac{\Delta_1}{\Delta_2}$ is

Ans : (2) Extremities of L.R. are (2,4) & (2, -4) $\Rightarrow \Delta_1 = 6$
 Points of intersection of tangents at these points are (-2, 0), (1, 3) & (-1, -1) $\Rightarrow \Delta_2 = 3$

67. Let $f(\theta) = \sin\left(\tan^{-1}\left(\frac{\sin \theta}{\sqrt{\cos 2\theta}}\right)\right)$, where $-\frac{\pi}{4} < \theta < \frac{\pi}{4}$. Then the value of $\frac{d}{d(\tan \theta)}(f(\theta))$ is .

Ans: (1) Put $k = \tan^{-1}\left(\frac{\sin \theta}{\sqrt{\cos 2\theta}}\right) \Rightarrow \tan k = \frac{\sin \theta}{\sqrt{\cos 2\theta}}$
 $\Rightarrow \tan^2 k = \frac{\sin^2 \theta}{1 - 2\sin^2 \theta} \Rightarrow \sec^2 k - 1 = \frac{1}{\operatorname{cosec}^2 \theta - 2}$
 $\Rightarrow \sec^2 k = \frac{\operatorname{cosec}^2 \theta - 1}{\operatorname{cosec}^2 \theta - 2} \Rightarrow \sin^2 k = \frac{1}{\cot^2 \theta} = \tan^2 \theta$
 $\Rightarrow \sin k = \tan \theta \therefore f(\theta) = \sin k = \tan \theta$ Hence, $\frac{d(f(\theta))}{d(\tan \theta)} = 1$

68. The minimum value of the sum of real numbers $a^{-5}, a^{-4}, 3a^{-3}, 1, a^8$ and a^{10} with $a > 0$ is

Ans: (8) Using A.M \geq G.M
 $\frac{a^{-5} + a^{-4} + a^{-3} + a^{-3} + a^{-3} + 1 + a^8 + a^{10}}{8} \geq 1$
 \therefore Minimum value of required expression = 8 (for $a = 1$).

69. The positive integer value of $n > 3$ satisfying the equation $\frac{1}{\sin\left(\frac{\pi}{n}\right)} = \frac{1}{\sin\left(\frac{2\pi}{n}\right)} + \frac{1}{\sin\left(\frac{3\pi}{n}\right)}$ is

69. (7) $\frac{1}{\sin x} = \frac{1}{\sin 2x} + \frac{1}{\sin 3x}$ { where $x = (\pi/n)$ }
 or $\frac{\sin 3x - \sin x}{\sin x \cdot \sin 3x} = \frac{1}{\sin 2x}$ or $\frac{2 \sin x \cdot \cos 2x}{\sin x \cdot \sin 3x} = \frac{1}{\sin 2x}$
 $\sin 4x = \sin 3x$ [$\because \sin x \neq 0$]
 $\therefore 4x = 3x$ (not possible $\because x \neq 0$)
 or $4x = \pi - 3x \Rightarrow 7x = \pi \Rightarrow 7 \cdot (\pi/n) = \pi \therefore n = 7$