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## IIT-JEE

### Previous Year

### Paper - II

### "2008"

### (Mathematics)

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## ***Mathematics***

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### PART - I

### SECTION - I

#### **Straight Objective Type**

This section contains 9 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

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1. An experiment has 10 equally likely outcomes. Let A and B be two non-empty events of the experiment. If A consists of 4 outcomes, the number of outcomes that B must have so that A and B are independent, is  
(A) 2, 4 or 8 (B) 3, 6 or 9

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- Sol.** (C) 4 or 8 (D) 5 or 10

$$P(A \cap B) = \frac{4}{10} \times \frac{p}{10} = \frac{2p}{5}$$

$$\Rightarrow \frac{2p}{5} \text{ is an integer}$$

$$\Rightarrow p = 5 \text{ or } 10.$$

2. The area of the region between the curves  $y = \sqrt{\frac{1+\sin x}{\cos x}}$  and  $y = \sqrt{\frac{1-\sin x}{\cos x}}$  bounded by the lines  $x = 0$  and  $x = \frac{\pi}{4}$  is

(A)  $\int_0^{\sqrt{2}-1} \frac{t}{(1+t^2)\sqrt{1-t^2}} dt$

(B)  $\int_0^{\sqrt{2}-1} \frac{4t}{(1+t^2)\sqrt{1-t^2}} dt$

(C)  $\int_0^{\sqrt{2}+1} \frac{4t}{(1+t^2)\sqrt{1-t^2}} dt$

(D)  $\int_0^{\sqrt{2}+1} \frac{t}{(1+t^2)\sqrt{1-t^2}} dt$

**Sol.** (B)

$$\begin{aligned} & \int_0^{\pi/4} \left( \sqrt{\frac{1+\sin x}{\cos x}} - \sqrt{\frac{1-\sin x}{\cos x}} \right) dx \\ &= \int_0^{\pi/4} \left( \sqrt{\frac{1+\tan \frac{x}{2}}{1-\tan \frac{x}{2}}} - \sqrt{\frac{1-\tan \frac{x}{2}}{1+\tan \frac{x}{2}}} \right) dx = \int \frac{\left(1+\tan \frac{x}{2}\right) - \left(1-\tan \frac{x}{2}\right)}{\sqrt{1-\tan^2 \frac{x}{2}}} dx \\ &= \int_0^{\pi/4} \frac{2 \tan \frac{x}{2}}{\sqrt{1-\tan^2 \frac{x}{2}}} dx = \int_0^{\sqrt{2}-1} \frac{4t}{(1+t^2)\sqrt{1-t^2}} dt \text{ as } \tan \frac{x}{2} = t. \end{aligned}$$

3. Consider three points  $P = (-\sin(\beta - \alpha), -\cos\beta)$ ,  $Q = (\cos(\beta - \alpha), \sin\beta)$  and  $R = (\cos(\beta - \alpha + \theta), \sin(\beta - \theta))$ , where  $0 < \alpha, \beta, \theta < \frac{\pi}{4}$ . Then

- (A) P lies on the line segment RQ  
(C) R lies on the line segment QP

- (B) Q lies on the line segment PR  
(D) P, Q, R are non-collinear

**Sol.** (D)

$$P \equiv (-\sin(\beta - \alpha), -\cos\beta) \equiv (x_1, y_1)$$

$$Q \equiv (\cos(\beta - \alpha), \sin\beta) \equiv (x_2, y_2)$$

$$\text{and } R \equiv (x_2 \cos\theta + x_1 \sin\theta, y_2 \cos\theta + y_1 \sin\theta)$$

$$\text{We see that } T \equiv \left( \frac{x_2 \cos\theta + x_1 \sin\theta}{\cos\theta + \sin\theta}, \frac{y_2 \cos\theta + y_1 \sin\theta}{\cos\theta + \sin\theta} \right)$$

and P, Q, T are collinear

$\Rightarrow$  P, Q, R are non-collinear.

4. Let  $I = \int \frac{e^x}{e^{4x} + e^{2x} + 1} dx$ ,  $J = \int \frac{e^{-x}}{e^{-4x} + e^{-2x} + 1} dx$ . Then, for an arbitrary constant C, the value of  $J - I$  equals

(A)  $\frac{1}{2} \log \left( \frac{e^{4x} - e^{2x} + 1}{e^{4x} + e^{2x} + 1} \right) + C$

(B)  $\frac{1}{2} \log \left( \frac{e^{2x} + e^x + 1}{e^{2x} - e^x + 1} \right) + C$

(C)  $\frac{1}{2} \log \left( \frac{e^{2x} - e^x + 1}{e^{2x} + e^x + 1} \right) + C$

(D)  $\frac{1}{2} \log \left( \frac{e^{4x} + e^{2x} + 1}{e^{4x} - e^{2x} + 1} \right) + C$

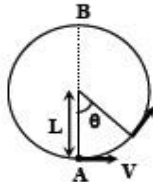
**Sol.** (C)

$$J - I = \int \frac{e^x(e^{2x} - 1)}{e^{4x} + e^{2x} + 1} dx = \int \frac{(z^2 - 1)}{z^4 + z^2 + 1} dz \quad \text{where } z = e^x$$

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25. A bob of mass  $M$  is suspended by a massless string of length  $L$ . The horizontal velocity  $V$  at position A is just sufficient to make it reach the point B. The angle  $\theta$  at which the speed of the bob is half of that at A, satisfies

Figure:



- (A)  $\theta = \frac{\pi}{4}$  (B)  $\frac{\pi}{4} < \theta < \frac{\pi}{2}$   
 (C)  $\frac{\pi}{2} < \theta < \frac{3\pi}{4}$  (D)  $\frac{3\pi}{4} < \theta < \pi$

Sol.

(D)

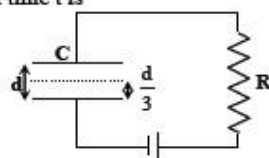
$$\frac{1}{2} 5mg\ell = \frac{1}{2} m \frac{5g\ell}{4} + mg\ell(1 - \cos \theta)$$

$$\cos \theta = -\frac{7}{8}$$

Hence,  $3\pi/4 < \theta < \pi$

26. A parallel plate capacitor C with plates of unit area and separation  $d$  is filled with a liquid of dielectric constant  $K = 2$ . The level of liquid is  $\frac{d}{3}$  initially. Suppose the liquid level decreases at a constant speed  $V$ , the time constant as a function of time  $t$  is

Figure:



- (A)  $\frac{6\epsilon_0 R}{5d + 3Vt}$  (B)  $\frac{(15d + 9Vt)\epsilon_0 R}{2d^2 - 3dVt - 9V^2t^2}$   
 (C)  $\frac{6\epsilon_0 R}{5d - 3Vt}$  (D)  $\frac{(15d - 9Vt)\epsilon_0 R}{2d^2 + 3dVt - 9V^2t^2}$

Sol.

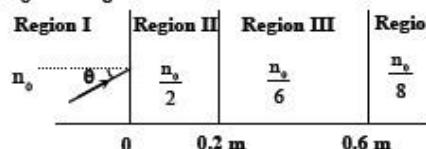
(A)

$$C_{\text{equivalent}} = \frac{\frac{2\epsilon_0}{d - vt} + \frac{\epsilon_0}{\frac{2d}{3} + vt}}{\frac{2\epsilon_0}{d - vt} + \frac{\epsilon_0}{\frac{2d}{3} + vt}}$$

$$\therefore \tau = C_{\text{equivalent}} R$$

27. A light beam is travelling from Region I to Region IV (Refer Figure). The refractive index in Regions I, II, III and IV are  $n_0$ ,  $\frac{n_0}{2}$ ,  $\frac{n_0}{6}$  and  $\frac{n_0}{8}$ , respectively. The angle of incidence  $\theta$  for which the beam just misses entering Region IV is

Figure:



- (A)  $\sin^{-1}\left(\frac{3}{4}\right)$  (B)  $\sin^{-1}\left(\frac{1}{8}\right)$  (C)  $\sin^{-1}\left(\frac{1}{4}\right)$  (D)  $\sin^{-1}\left(\frac{1}{3}\right)$

**Sol.** (B)

Total internal reflection occurs at the interface of region III and IV.

Because mediums are parallel

$$n_0 \sin \theta = \frac{n_0}{8} \sin \left( \frac{\pi}{2} \right)$$

$$\sin \theta = 1/8$$

28. A vibrating string of certain length  $\ell$  under a tension  $T$  resonates with a mode corresponding to the first overtone (third harmonic) of an air column of length 75 cm inside a tube closed at one end. The string also generates 4 beats per second when excited along with a tuning fork of frequency  $n$ . Now when the tension of the string is slightly increased the number of beats reduces 2 per second. Assuming the velocity of sound in air to be 340 m/s, the frequency  $n$  of the tuning fork in Hz is

- (A) 344 (B) 336 (C) 117.3 (D) 109.3

**Sol.** (A)

$$n_s = \frac{3}{4} \left( \frac{340}{0.75} \right) = n - 4$$

$$\therefore n = 344 \text{ Hz}$$

29. A radioactive sample S1 having an activity  $5\mu\text{Ci}$  has twice the number of nuclei as another sample S2 which has an activity of  $10\mu\text{Ci}$ . The half lives of S1 and S2 can be

- (A) 20 years and 5 years, respectively  
 (B) 20 years and 10 years, respectively  
 (C) 10 years each  
 (D) 5 years each

**Sol.** (A)

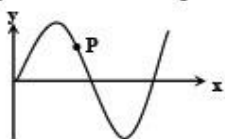
$$5\mu\text{Ci} = \frac{\ln 2}{T_1} (2N_0)$$

$$10\mu\text{Ci} = \frac{\ln 2}{T_2} (N_0)$$

$$\text{Dividing we get } T_1 = 4T_2$$

30. A transverse sinusoidal wave moves along a string in the positive  $x$ -direction at a speed of 10 cm/s. The wavelength of the wave is 0.5 m and its amplitude is 10 cm. At a particular time  $t$ , the snap-shot of the wave is shown in figure. The velocity of point P when its displacement is 5 cm is

Figure:



(A)  $\frac{\sqrt{3}\pi}{50} \text{ j m/s}$

(B)  $-\frac{\sqrt{3}\pi}{50} \text{ j m/s}$

(C)  $\frac{\sqrt{3}\pi}{50} \text{ i m/s}$

(D)  $-\frac{\sqrt{3}\pi}{50} \text{ i m/s}$

**Sol.** (A)

$$y = 5 \text{ cm and } V = +ve$$

$$y = A \sin(\omega t \pm \phi) \quad V = A\omega \cos(\omega t \pm \phi)$$

$$\text{We get } \omega t \pm \phi = 30^\circ$$

$$\omega = 2\pi \frac{v}{\lambda} = \frac{2\pi}{5}$$

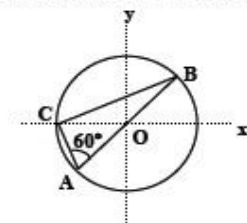
$$v = A\omega \cos(\omega t \pm \phi) = \left( \frac{10}{100} \right) \times \left( \frac{2\pi}{5} \right) \left( \frac{\sqrt{3}}{2} \right) = \frac{\pi\sqrt{3}}{50} \text{ m/s}$$

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31. Consider a system of three charges  $\frac{q}{3}$ ,  $\frac{q}{3}$  and  $-\frac{2q}{3}$  placed at points A, B and C, respectively, as shown in the figure.

Take O to be the centre of the circle of radius R and angle CAB = 60°

Figure:



(A) The electric field at point O is  $\frac{q}{8\pi\epsilon_0 R^2}$  directed along the negative x-axis

(B) The potential energy of the system is zero

(C) The magnitude of the force between the charges at C and B is  $\frac{q^2}{54\pi\epsilon_0 R^2}$

(D) The potential at point O is  $\frac{q}{12\pi\epsilon_0 R}$

Sol. (C)

$$F_{BC} = \frac{1}{4\pi\epsilon_0} \frac{\left(\frac{q}{3}\right)\left(\frac{2q}{3}\right)}{\left(R\sqrt{3}\right)^2} = \frac{q^2}{54\pi\epsilon_0 R^2}$$

## SECTION – II

### Reasoning Type

This section contains 4 reasoning type questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

32. STATEMENT-1  
For practical purposes, the earth is used as a reference at zero potential in electrical circuits.

and

STATEMENT-2

The electrical potential of a sphere of radius R with charge Q uniformly distributed on the surface is given by  $\frac{Q}{4\pi\epsilon_0 R}$ .

- (A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is a correct explanation for STATEMENT-1  
 (B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is NOT a correct explanation for STATEMENT-1  
 (C) STATEMENT -1 is True, STATEMENT-2 is False  
 (D) STATEMENT -1 is False, STATEMENT-2 is True

Sol.

(B)

33. STATEMENT-1  
It is easier to pull a heavy object than to push it on a level ground.

and

STATEMENT-2

The magnitude of frictional force depends on the nature of the two surfaces in contact.

- (A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is a correct explanation for STATEMENT-1  
 (B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is NOT a correct explanation for STATEMENT-1  
 (C) STATEMENT -1 is True, STATEMENT-2 is False  
 (D) STATEMENT -1 is False, STATEMENT-2 is True

**Sol.** (B)  
In pushing Normal contact force is greater than in pulling.

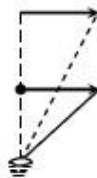
34 STATEMENT-1  
For an observer looking out through the window of a fast moving train, the nearby objects appear to move in the opposite direction to the train, while the distant objects appear to be stationary.

and

STATEMENT-2  
If the observer and the object are moving at velocities  $\vec{V}_1$  and  $\vec{V}_2$  respectively with reference to a laboratory frame, the velocity of the object with respect to the observer is  $\vec{V}_2 - \vec{V}_1$ .

- (A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is a correct explanation for STATEMENT-1  
 (B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is NOT a correct explanation for STATEMENT-1  
 (C) STATEMENT -1 is True, STATEMENT-2 is False  
 (D) STATEMENT -1 is False, STATEMENT-2 is True

**Sol.** (B)  
Distance appeared to move depends upon angle subtended on eye.



35 STATEMENT-1  
The sensitivity of a moving coil galvanometer is increased by placing a suitable magnetic material as a core inside the coil.

and

STATEMENT-2  
Soft iron has a high magnetic permeability and cannot be easily magnetized or demagnetized.

- (A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is a correct explanation for STATEMENT-1  
 (B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is NOT a correct explanation for STATEMENT-1  
 (C) STATEMENT -1 is True, STATEMENT-2 is False  
 (D) STATEMENT -1 is False, STATEMENT-2 is True

**Sol.** (C)

SECTION – III

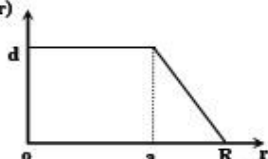
Linked Comprehension Type

This section contains 2 paragraphs. Based upon each paragraph, 3 multiple choice questions have to be answered. Each question has 4 choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

Paragraph for Question Nos. 36 to 38

The nuclear charge ( $Ze$ ) is non-uniformly distributed within a nucleus of radius  $R$ . The charge density  $\rho(r)$  [charge per unit volume] is dependent only on the radial distance  $r$  from the centre of the nucleus as shown in figure. The electric field is only along the radial direction.

Figure:  $\rho(r)$



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36. The electric field at  $r = R$  is  
 (A) independent of  $a$  (B) directly proportional to  $a$   
 (C) directly proportional to  $a^2$  (D) inversely proportional to  $a$

Sol. (A)

37. For  $a = 0$ , the value of  $d$  (maximum value of  $\rho$  as shown in the figure) is  
 (A)  $\frac{3Ze}{4\pi R^3}$  (B)  $\frac{3Ze}{\pi R^3}$  (C)  $\frac{4Ze}{3\pi R^3}$  (D)  $\frac{Ze}{3\pi R^3}$

Sol. (B)  

$$q = \int_0^R \frac{d}{R}(R-x)4\pi x^2 dx = Ze$$

$$d = \frac{3Ze}{\pi R^3}$$

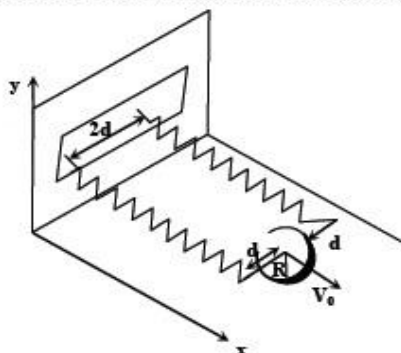
38. The electric field within the nucleus is generally observed to be linearly dependent on  $r$ . This implies.  
 (A)  $a = 0$  (B)  $a = \frac{R}{2}$  (C)  $a = R$  (D)  $a = \frac{2R}{3}$

Sol. (C)  
 If within a sphere  $\rho$  is constant  $E \propto r$

Paragraph for Question Nos. 39 to 41

A uniform thin cylindrical disk of mass  $M$  and radius  $R$  is attached to two identical massless springs of spring constant  $k$  which are fixed to the wall as shown in the figure. The springs are attached to the axle of the disk symmetrically on either side at a distance  $d$  from its centre. The axle is massless and both the springs and the axle are in horizontal plane. The unstretched length of each spring is  $L$ . The disk is initially at its equilibrium position with its centre of mass (CM) at a distance  $L$  from the wall. The disk rolls without slipping with velocity  $\vec{V}_0 = V_0 \hat{i}$ . The coefficient of friction is  $\mu$ .

Figure:



39. The net external force acting on the disk when its centre of mass is at displacement  $x$  with respect to its equilibrium position is  
 (A)  $-kx$  (B)  $-2kx$  (C)  $-\frac{2kx}{3}$  (D)  $-\frac{4kx}{3}$

Sol. (D)  
 $2kx - f = ma$   
 $\Rightarrow fR = I\alpha$   
 $a = R\alpha$   
 $\Rightarrow ma = \frac{4kx}{3}$

40. The centre of mass of the disk undergoes simple harmonic motion with angular frequency  $\omega$  equal to  
 (A)  $\sqrt{\frac{k}{M}}$  (B)  $\sqrt{\frac{2k}{M}}$  (C)  $\sqrt{\frac{2k}{3M}}$  (D)  $\sqrt{\frac{4k}{3M}}$



*Sol.* (D)  
 $-(2kx)R = I_p \alpha$   
 $\alpha = -\frac{4kR}{3mR^2}(R\theta) = -\frac{4k}{3m}\theta$

41. The maximum value of  $V_0$  for which the disk will roll without slipping is  
 (A)  $\mu g \sqrt{\frac{M}{k}}$  (B)  $\mu g \sqrt{\frac{M}{2k}}$  (C)  $\mu g \sqrt{\frac{3M}{k}}$  (D)  $\mu g \sqrt{\frac{5M}{2k}}$

*Sol.* (C)  
 $2kx - f_{\max} = ma$   
 $2kx \cdot r = I_p \alpha$   
 $f_{\max} = \mu mg$   
 $\Rightarrow x = \frac{3}{2} \frac{\mu mg}{k}$   
 $\Rightarrow \frac{1}{2}(2k)x^2 = \frac{1}{2}I_p \omega^2$   
 $\Rightarrow \gamma = \mu g \sqrt{\frac{3m}{k}}$

SECTION – IV

Matrix-Match Type

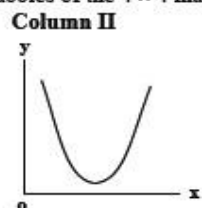
This section contains 3 questions. Each question contains statements given in two columns which have to be matched. Statements in Column I are labelled as A, B, C and D whereas statements in Column II are labelled as p, q, r and s. The answers to these questions have to be appropriately bubbled as illustrated in the following example.

If the correct matches are A-p, A-r, B-p, B-s, C-r, C-s and D-q, then the correctly bubbled matrix will be look like the following:

	p	q	r	s
A	<input type="radio"/>	<input checked="" type="radio"/>	<input checked="" type="radio"/>	<input type="radio"/>
B	<input checked="" type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input checked="" type="radio"/>
C	<input type="radio"/>	<input type="radio"/>	<input checked="" type="radio"/>	<input checked="" type="radio"/>
D	<input type="radio"/>	<input checked="" type="radio"/>	<input type="radio"/>	<input type="radio"/>

42. Column I gives a list of possible set of parameters measured in some experiments. The variations of the parameters in the form of graphs are shown in Column II. Match the set of parameters given in Column I with the graph given in Column II. Indicate your answer by darkening the appropriate bubbles of the  $4 \times 4$  matrix given in the ORS.

**Column I**  
 (A) Potential energy of a simple pendulum (y axis) as a function of displacement (x axis)



(B) Displacement (y axis) as a function of time (x axis) for a one dimensional motion at zero or constant acceleration when the body is moving along the positive x-direction

