Download IIT-JEE Previous Year Paper - II "2010" (Mathematics)

PART - II: MATHEMATICS

SECTION – I (Single Correct Choice Type)

This Section contains **6 multiple choice questions.** Each question has four choices A), B), C) and D) out of which **ONLY ONE** is correct.

20. If the distance of the point P (1, – 2, 1) from the plane x + 2y – 2z = α, where α > 0, is 5, then the foot of the perpendicular from P to the plane is

(A) $\left(\frac{8}{3}, \frac{4}{3}, -\frac{7}{3}\right)$ (B) $\left(\frac{4}{3}, -\frac{4}{3}, \frac{1}{3}\right)$

(C) $\left(\frac{1}{3}, \frac{2}{3}, \frac{10}{3}\right)$ (D) $\left(\frac{2}{3}, -\frac{1}{3}, \frac{5}{2}\right)$

Sol. ***(A)***

Distance of point (1, – 2, 1) from plane x + 2y – 2z = α is 5 ⇒ α = 10.

Equation of PQ $\frac{x-1}{1}=\frac{y+2}{2}=\frac{z-1}{-2}=t$

$Q = (t + 1, 2t - 2, -2t + 1)$ and $PQ = 5 \Rightarrow t = \frac{5+\alpha}{9}=\frac{5}{3} \Rightarrow Q = \left(\frac{8}{3}, \frac{4}{3}, \frac{-7}{3}\right)$.

21. A signal which can be green or red with probability $\frac{4}{5}$ and $\frac{1}{5}$ respectively, is received by station A and then transmitted to station B. The probability of each station receiving the signal correctly is $\frac{3}{4}$. If the signal received at station B is green, then the probability that the original signal was green is

(A) $\frac{3}{5}$ (B) $\frac{6}{7}$

(C) $\frac{20}{23}$ (D) $\frac{9}{20}$

Sol. ***(C)***

Event G = original signal is green

E_1 = A receives the signal correct

E_2 = B receives the signal correct

E = signal received by B is green

P(signal received by B is green) = $P(GE_1E_2) + P(G\bar{E}_1\bar{E}_2)+P(\bar{G}E_1\bar{E}_2)+P(\bar{G}\bar{E}_1E_2)$

$P(E) = \frac{46}{5\times16}$

$P(G/E) = \frac{40/5\times16}{46/5\times16}=\frac{20}{23}$.

22. Two adjacent sides of a parallelogram ABCD are given by $\overline{AB} = 2\hat{i}+10\hat{j}+11\hat{k}$ and $\overline{AD} = -\hat{i}+2\hat{j}+2\hat{k}$. The side AD is rotated by an acute angle α in the plane of the parallelogram so that AD becomes AD′. If AD′ makes a right angle with the side AB, then the cosine of the angle α is given by

(A) $\frac{8}{9}$ (B) $\frac{\sqrt{17}}{9}$

(C) $\frac{1}{9}$ (D) $\frac{4\sqrt{5}}{9}$

Sol. *(B)*

$\overline{AD} = \overline{AB}\times(\overline{AB}\times\overline{AD}) = 5(61\hat{i} - 10\hat{j} - 21\hat{k}) \Rightarrow \cos\alpha = \frac{|\overline{AD}'\cdot\overline{AD}|}{|\overline{AD}'||\overline{AD}|} = \frac{\sqrt{17}}{9}$.

*23. For r = 0, 1, …, 10, let A_r, B_r and C_r denote, respectively, the coefficient of x^r in the expansions of $(1 + x)^{10}$, $(1 + x)^{20}$ and $(1 + x)^{30}$. Then $\sum_{r=1}^{10} A_r(B_{10}B_r - C_{10}A_r)$ is equal to

(A) $B_{10} - C_{10}$
(B) $A_{10}(B_{10}^2 - C_{10}A_{10})$
(C) 0
(D) $C_{10} - B_{10}$

Sol. *(D)*

Let $y = \sum_{r=1}^{10} A_r(B_{10}B_r - C_{10}A_r)$

$\sum_{r=1}^{10} A_rB_r$ = coefficient of x^{20} in $((1 + x)^{10}(x + 1)^{20}) - 1$

$= C_{20} - 1 = C_{10} - 1$ and $\sum_{r=1}^{10}(A_r)^2$ = coefficient of x^{10} in $((1 + x)^{10}(x + 1)^{10}) - 1 = B_{10} - 1$

$\Rightarrow y = B_{10}(C_{10} - 1) - C_{10}(B_{10} - 1) = C_{10} - B_{10}$.

24. Let f be a real-valued function defined on the interval (– 1, 1) such that $e^{-x}f(x) = 2 + \int_0^x \sqrt{t^4+1}\,dt$, for all x $\in (-1, 1)$ and let f^{-1} be the inverse function of f. Then $(f^{-1})'(2)$ is equal to

(A) 1
(B) 1/3
(C) 1/2
(D) 1/e

Sol. *(B)*

$e^{-x}f(x) = 2 + \int_0^x \sqrt{t^4+1}\,dt \quad \ldots(i)$

$f(f^{-1}(x)) = x$

$\Rightarrow f'(f^{-1}(x))\,(f^{-1}(x))' = 1 \Rightarrow (f^{-1}(2))' = \frac{1}{f'(f^{-1}(2))} \Rightarrow f(0) = 2 \Rightarrow f^{-1}(2) = 0$

$(f^{-1}(2))' = \frac{1}{f'(0)}$

$e^{-x}(f'(x) - f(x)) = \sqrt{x^4+1}$

Put $x = 0 \Rightarrow f'(0) - 2 = 1 \Rightarrow f'(0) = 3$

$(f^{-1}(2))' = 1/3$.

*25. Let S = {1, 2, 3, 4}. The total number of unordered pairs of disjoint subsets of S is equal to

(A) 25
(B) 34
(C) 42
(D) 41

Sol. *(D)*

Total number of unordered pairs of disjoint subsets

$= \frac{3^4+1}{2} = 41$.

SECTION – II (Integer Type)

This Section contains **5 questions.** The answer to each question is a **single-digit integer,** ranging from 0 to 9. The correct digit below the question no. in the **ORS** is to be bubbled.

*26. Let $a_1, a_2, a_3, \ldots, a_{11}$ be real numbers satisfying $a_1 = 15$, $27 - 2a_2 > 0$ and $a_k = 2a_{k-1} - a_{k-2}$ for k = 3, 4, …, 11. If $\frac{a_1^2 + a_2^2 + \ldots + a_{11}^2}{11} = 90$, then the value of $\frac{a_1 + a_2 + \ldots + a_{11}}{11}$ is equal to

Sol. ***(0)***

$a_k = 2a_{k-1} - a_{k-2} \Rightarrow a_1, a_2, \ldots, a_{11}$ are in A.P.

$\therefore \frac{a_1^2 + a_2^2 + \ldots + a_{11}^2}{11} = \frac{11a^2 + 35\times 11d^2 + 10ad}{11} = 90$

$\Rightarrow 225 + 35d^2 + 150d = 90$

$35d^2 + 150d + 135 = 0 \Rightarrow d = -3, -9/7$

Given $a_2 < \frac{27}{2}$ $\therefore$ $d = -3$ and $d \neq -9/7 \Rightarrow \frac{a_1 + a_2 + \ldots + a_{11}}{11} = \frac{11}{2}[30 - 10\times 3] = 0$.

27. Let f be a function defined on R (the set of all real numbers) such that $f'(x) = 2010(x - 2009)(x - 2010)^2 (x - 2011)^3 (x - 2012)^4$, for all $x \in R$. If g is a function defined on R with values in the interval $(0, \infty)$ such that $f(x) = \ln(g(x))$, for all $x \in R$, then the number of points in R at which g has a local maximum is

Sol. ***(1)***

$f(x) = \ln\{g(x)\}$

$g(x) = e^{f(x)}$

$g'(x) = e^{f(x)} . f'(x)$

$g'(x) = 0 \Rightarrow f'(x) = 0$ as $e^{f(x)} \neq 0$

$\Rightarrow 2010(x - 2009)(x - 2010)^2(x - 2011)^3(x - 2012)^4 = 0$

so there is only one point of local maxima.

28. Let k be a positive real number and let $A = \begin{bmatrix} 2k-1 & 2\sqrt{k} & 2\sqrt{k} \\ 2\sqrt{k} & 1 & -2k \\ -2\sqrt{k} & 2k & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 2k-1 & \sqrt{k} \\ 1-2k & 0 & 2\sqrt{k} \\ -\sqrt{k} & -2\sqrt{k} & 0 \end{bmatrix}$. If det (adj A) + det (adj B) = 10^6, then [k] is equal to

[Note : adj M denotes the adjoint of a square matrix M and [k] denotes the largest integer less than or equal to k].

Sol. ***(5)***

$|A| = (2k + 1)^3$, $|B| = 0$ (Since B is a skew–symmetric matrix of order 3)

$\Rightarrow$ det (adj A) $= |A|^{n-1} = ((2k + 1)^3)^2 = 106 \Rightarrow 2k + 1 = 10 \Rightarrow 2k = 9$

[k] = 4.

*29. Two parallel chords of a circle of radius 2 are at a distance $\sqrt{3} + 1$ apart. If the chords subtend at the centre, angles of $\frac{\pi}{k}$ and $\frac{2\pi}{k}$, where k > 0, then the value of [k] is

[Note : [k] denotes the largest integer less than or equal to k].

Sol. ***(2)***

$2\cos\frac{\pi}{2k} + 2\cos\frac{\pi}{k} = \sqrt{3} + 1$

$$\cos\frac{\pi}{2k}+\cos\frac{\pi}{k}=\frac{\sqrt{3}+1}{2}$$

Let $\frac{\pi}{k}=\theta$, $\cos\theta+\cos\frac{\theta}{2}=\frac{\sqrt{3}+1}{2} \Rightarrow 2\cos^2\frac{\theta}{2}-1+\cos\frac{\theta}{2}=\frac{\sqrt{3}+1}{2}$

$\cos\frac{\theta}{2}=t \qquad 2t^2+t-\frac{\sqrt{3}+3}{2}=0$

$t=\frac{-1\pm\sqrt{1+4(3+\sqrt{3})}}{4}=\frac{-1\pm(2\sqrt{3}+1)}{4}=\frac{-2-2\sqrt{3}}{4},\ \frac{\sqrt{3}}{2} \qquad \because t\in[-1,1],\ \cos\frac{\theta}{2}=\frac{\sqrt{3}}{2}$

$\frac{\theta}{2}=\frac{\pi}{6}\Rightarrow k=3.$

*30. Consider a triangle ABC and let a, b and c denote the lengths of the sides opposite to vertices A, B and C respectively. Suppose a = 6, b = 10 and the area of the triangle is $15\sqrt{3}$. If $\angle ACB$ is obtuse and if r denotes the radius of the incircle of the triangle, then r^2 is equal to

Sol. *(3)*

$\Delta=\frac{1}{2}ab\sin C \Rightarrow \sin C=\frac{2\Delta}{ab}=\frac{2\times15\sqrt{3}}{6\times10}=\frac{\sqrt{3}}{2} \Rightarrow C=120°$

$\Rightarrow c=\sqrt{a^2+b^2-2ab\cos C}$

$=\sqrt{6^2+10^2-2\times6\times10\times\cos120°}=14$

$\therefore r=\frac{\Delta}{s}\Rightarrow r^2=\frac{225\times3}{\left(\frac{6+10+14}{2}\right)^2}=3.$

SECTION – III (Paragraph Type)

This Section contains **2 paragraphs.** Based upon each of the paragraphs **3 multiple choice questions** have to be answered. Each of these questions has four choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

Paragraph for questions 31 to 33.

Consider the polynomial $f(x) = 1 + 2x + 3x^2 + 4x^3$. Let s be the sum of all distinct real roots of f(x) and let t = |s|.

31. The real number s lies in the interval

(A) $\left(-\frac{1}{4},\ 0\right)$ (B) $\left(-11,\ -\frac{3}{4}\right)$

(C) $\left(-\frac{3}{4},\ -\frac{1}{2}\right)$ (D) $\left(0,\ \frac{1}{4}\right)$

Sol. *(C)*

Since, $f\left(-\frac{1}{2}\right)\cdot f\left(-\frac{3}{4}\right)<0 \Rightarrow$ S lie in $\left(-\frac{3}{4},\ -\frac{1}{2}\right)$.

32. The area bounded by the curve y = f(x) and the lines x = 0, y = 0 and x = t, lies in the interval

(A) $\left(\frac{3}{4},\ 3\right)$ (B) $\left(\frac{21}{64},\ \frac{11}{16}\right)$

(C) (9, 10) (D) $\left(0,\ \frac{21}{64}\right)$

$\cos\frac{\pi}{2k}+\cos\frac{\pi}{k}=\frac{\sqrt{3}+1}{2}$

Let $\frac{\pi}{k}=0$, $\cos\theta+\cos\frac{\theta}{2}=\frac{\sqrt{3}+1}{2} \Rightarrow 2\cos^2\frac{\theta}{2}-1+\cos\frac{\theta}{2}=\frac{\sqrt{3}+1}{2}$

$\cos\frac{\theta}{2}=t \qquad 2t^2+t-\frac{\sqrt{3}+3}{2}=0$

$t=\frac{-1\pm\sqrt{1+4(3+\sqrt{3})}}{4}=\frac{-1\pm(2\sqrt{3}+1)}{4}=\frac{-2-2\sqrt{3}}{4},\frac{\sqrt{3}}{2} \quad \because t\in[-1,1],\ \cos\frac{\theta}{2}=\frac{\sqrt{3}}{2}$

$\frac{\theta}{2}=\frac{\pi}{6}\Rightarrow k=3.$

*30. Consider a triangle ABC and let a, b and c denote the lengths of the sides opposite to vertices A, B and C respectively. Suppose a = 6, b = 10 and the area of the triangle is $15\sqrt{3}$. If $\angle ACB$ is obtuse and if r denotes the radius of the incircle of the triangle, then r^2 is equal to

Sol. *(3)*

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$\Rightarrow c=\sqrt{a^2+b^2-2ab\cos C}$

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Sol. *(C)*

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Sol. ***(A)***

$-\frac{3}{4} < s < -\frac{1}{2}$

$\frac{1}{2} < t < \frac{3}{4}$

$\int_0^{1/2} (4x^3 + 3x^2 + 2x + 1)dx < \text{area} < \int_0^{3/4} (4x^3 + 3x^2 + 2x + 1)\,dx$

$[x^4 + x^3 + x^2 + x]_0^{1/2} < area < [x^4 + x^3 + x^2 + x]_0^{3/4}$

$\frac{1}{16} + \frac{1}{8} + \frac{1}{4} + \frac{1}{2} < area < \frac{81}{256} + \frac{27}{64} + \frac{9}{16} + \frac{3}{4}$

$\frac{15}{16} < area < \frac{525}{256}$.

33. The function f′(x) is

(A) increasing in $\left(-t, -\frac{1}{4}\right)$ and decreasing in $\left(-\frac{1}{4}, t\right)$

(B) decreasing in $\left(-t, -\frac{1}{4}\right)$ and increasing in $\left(-\frac{1}{4}, t\right)$

(C) increasing in (–t, t)

(D) decreasing in (–t, t)

Sol. ***(B)***

$f''(x) = 2[12x + 3] = 0 \Rightarrow x = -1/4$.

Paragraph for questions 34 to 36.

Tangents are drawn from the point P(3, 4) to the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ touching the ellipse at points A and B.

*34. The coordinates of A and B are

(A) (3, 0) and (0, 2)

(B) $\left(-\frac{8}{5}, \frac{2\sqrt{161}}{15}\right)$ and $\left(-\frac{9}{5}, \frac{8}{5}\right)$

(C) $\left(-\frac{8}{5}, \frac{2\sqrt{161}}{15}\right)$ and (0, 2)

(D) (3, 0) and $\left(-\frac{9}{5}, \frac{8}{5}\right)$

Sol. ***(D)***

$y = mx + \sqrt{9m^2 + 4}$

$4 - 3m = \sqrt{9m^2 + 4}$

$16 + 9m^2 - 24m = 9m^2 + 4 \Rightarrow m = \frac{12}{24} = \frac{1}{2}$

Equation is $y - 4 = \frac{1}{2}(x - 3)$

$2y - 8 = x - 3 \Rightarrow x - 2y + 5 = 0$

Let B = (α, β) $\Rightarrow \frac{x\alpha}{9} + \frac{y\beta}{4} - 1 = 0 \Rightarrow \frac{\alpha/9}{1} = \frac{\beta/4}{-2} = \frac{-1}{5} \Rightarrow \alpha = -\frac{9}{5}, \beta = \frac{8}{5}$

$B \equiv \left(-\frac{9}{5}, \frac{8}{5}\right)$.

*35. The orthocentre of the triangle PAB is

(A) $\left(5, \frac{8}{7}\right)$ (B) $\left(\frac{7}{5}, \frac{25}{8}\right)$

(C) $\left(\frac{11}{5}, \frac{8}{5}\right)$ (D) $\left(\frac{8}{25}, \frac{7}{5}\right)$

Sol. *(C)*

Slope of BD must be 0

$\Rightarrow y - \frac{8}{5} = 0 \left(x + \frac{9}{5}\right) \Rightarrow y = \frac{8}{5}$

Hence y coordinate of D is 8/5.

*36. The equation of the locus of the point whose distances from the point P and the line AB are equal, is

(A) $9x^2 + y^2 - 6xy - 54x - 62y + 241 = 0$ (B) $x^2 + 9y^2 + 6xy - 54x + 62y - 241 = 0$

(C) $9x^2 + 9y^2 - 6xy - 54x - 62y - 241 = 0$ (D) $x^2 + y^2 - 2xy + 27x + 31y - 120 = 0$

Sol. *(A)*

Locus is parabola

Equation of AB Is $\frac{3x}{9} + \frac{4y}{4} = 1 \Rightarrow \frac{x}{3} + y = 1 \Rightarrow x + 3y - 3 = 0$

$(x-3)^2 + (y-4)^2 = \frac{(x+3y-3)^2}{10}$

$10x^2 + 90 - 60x + 10y^2 + 160 - 80y = x^2 + 9y^2 + 9 + 6xy - 6x - 18y$

$\Rightarrow 9x^2 + y^2 - 6xy - 54x - 62y + 241 = 0.$

SECTION – IV (Matrix Type)

This Section contains **2 questions.** Each question has **four statements** (A, B, C and D) given in **Column I** and five statements (p, q, r, s and t) in **Column II**. Any given statement in **Column I** can have correct matching with one or more statement(s) given in **Column II**. For example, if for a given question, statement B matches with the statements given in q and r, then for that particular question, against statement B, darken the bubbles corresponding to q and r in the **ORS**.

*37. Match the statements in column-I with those in column-II.

[Note: Here z takes the values in the complex plane and Im z and Re z denote, respectively, the imaginary part and the real part of z]

	Column – I		**Column – II**
(A)	The set of points z satisfying $\|z - i\|z\|\| = \|z + i\|z\|\|$ is contained in or equal to	(p)	an ellipse with eccentricity $\frac{4}{5}$
(B)	The set of points z satisfying $\|z + 4\| + \|z - 4\| = 10$ is contained in or equal to	(q)	the set of points z satisfying Im z = 0
(C)	If $\|\omega\| = 2$, then the set of points $z = \omega - 1/\omega$ is contained in or equal to	(r)	the set of points z satisfying $\|\text{Im } z\| \le 1$
(D)	If $\|\omega\| = 1$, then the set of points $z = \omega + 1/\omega$ is contained in or equal to	(s)	the set of points z satisfying $\|\text{Re } z\| \le 1$
		(t)	the set of points z satisfying $\|z\| \le 3$

Sol. (A) (q)

$\left|\frac{z}{|z|} - i\right| = \left|\frac{z}{|z|} + i\right|, z \neq 0$

$\frac{z}{|z|}$ is unimodular complex number

and lies on perpendicular bisector of i and –i

$\Rightarrow \frac{z}{|z|} = \pm 1 \Rightarrow z = \pm 1|z| \Rightarrow$ a is real number $\Rightarrow \text{Im}(z) = 0$.

(B) ***(p)***

$|z + 4| + |z - 4| = 10$

z lies on an ellipse whose focus are (4, 0) and (–4, 0) and length of major axis is 10

$\Rightarrow 2ae = 8$ and $2a = 10 \Rightarrow e = 4/5$

$|\text{Re}(z)| \le 5$.

(C) ***(p), (t)***

$|w| = 2 \Rightarrow w = 2(\cos\theta + i\sin\theta)$

$x + iy = 2(\cos\theta + i\sin\theta) - \frac{1}{2}(\cos\theta - i\sin\theta)$

$= \frac{3}{2}\cos\theta + i\frac{5}{2}\sin\theta \Rightarrow \frac{x^2}{(3/2)^2} + \frac{y^2}{(5/2)^2} = 1$

$e^2 = 1 - \frac{9/4}{25/4} = 1 - \frac{9}{25} = \frac{16}{25} \Rightarrow e = \frac{4}{5}$

(D) ***(q), (t)***

$|w| = 1 \Rightarrow x + iy = \cos + i\sin\theta + \cos\theta - i\sin\theta$

$x + iy = 2\cos\theta$

$|\text{Re}(z)| \le 1$, $|\text{Im}(z) = 0$.

38. Match the statements in column-I with those in column-II.

	Column – I		Column – II
(A)	A line from the origin meets the lines $\frac{x-2}{1} = \frac{y-1}{-2} = \frac{z+1}{1}$ and $\frac{x-\frac{8}{3}}{2} = \frac{y+3}{-1} = \frac{z-1}{1}$ at P and Q respectively. If length PQ = d, then d^2 is	(p)	– 4
*(B)	The values of x satisfying $\tan^{-1}(x + 3) - \tan^{-1}(x - 3) = \sin^{-1}\left(\frac{3}{5}\right)$ are	(q)	0
(C)	Non-zero vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ satisfy $\vec{a}\cdot\vec{b} = 0$, $(\vec{b}-\vec{a})\cdot(\vec{b}+\vec{c}) = 0$ and $2\|\vec{b}+\vec{c}\| = \|\vec{b}-\vec{a}\|$. If $\vec{a} = \mu\vec{b} + 4\vec{c}$, then the possible values of μ are	(r)	4
(D)	Let f be the function on [–π, π] given by f(0) = 9 and f(x) = $\sin\left(\frac{9x}{2}\right)\Big/\sin\left(\frac{x}{2}\right)$ for x ≠ 0. The value of $\frac{2}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$ is	(s)	5
		(t)	6

Sol. (A). (t)

Let the line be $\frac{x}{a} = \frac{y}{b} = \frac{z}{c}$ intersects the lines

$\Rightarrow$ S. D = 0 $\Rightarrow a + 3b + 5c = 0$ and $3a + b - 5c = 0 \Rightarrow a : b : c :: 5r : -5r : 2r$

on solving with given lines we get points of intersection P ≡ (5, – 5, 2) and Q ≡ $\left(\frac{10}{3}, -\frac{10}{3}, \frac{8}{3}\right)$

$\Rightarrow PQ^2 = d^2 = 6.$

(B) **(p), (r)**

$\tan^{-1}(x+3) - \tan^{-1}(x-3) = \sin^{-1}(3/5)$

$\Rightarrow \tan^{-1}\frac{(x+3)-(x-3)}{1+(x^2-9)} = \tan^{-1}\frac{3}{4} \Rightarrow \frac{6}{x^2-8} = \frac{3}{4}$

$\therefore x^2 - 8 = 8$

or $x = \pm 4$.

(C) **(q), (s)**

As $\vec{a} = \mu\vec{b} + 4\vec{c} \Rightarrow \mu(|\vec{b}|) = -4\vec{b}\cdot\vec{c}$ and $|\vec{b}|^2 = 4\vec{a}\cdot\vec{c}$ and $|\vec{b}|^2 + \vec{b}\cdot\vec{c} - \vec{a}\cdot\vec{c} = 0$

Again, as $2|\vec{b}+\vec{c}| = |\vec{b}-\vec{a}|$

Solving and eliminating $\vec{b}\cdot\vec{c}$ and eliminating $|\vec{a}|^2$

we get $(2\mu^2 - 10\mu)|\vec{b}|^2 = 0 \Rightarrow \mu = 0$ and 5.

(D) **(r)**

$$I = \frac{2}{\pi}\int_{-\pi}^{\pi}\frac{\sin 9(x/2)}{\sin(x/2)}dx = \frac{2}{\pi}\times 2\int_0^{\pi}\frac{\sin 9(x/2)}{\sin(x/2)}dx$$

$x/2 = \theta \Rightarrow dx = 2d\theta$

$x = 0, \theta = 0$

$x = \pi\ \theta = \pi/2$

$$I = \frac{8}{\pi}\int_0^{\pi/2}\frac{\sin 9\theta}{\sin\theta}d\theta$$

$$= \frac{8}{\pi}\int_0^{\pi/2}\frac{(\sin 9\theta - \sin 7\theta)}{\sin\theta} + \frac{(\sin 7\theta - \sin 5\theta)}{\sin\theta} + \frac{(\sin 5\theta - \sin 3\theta)}{\sin\theta} + \frac{(\sin 3\theta - \sin\theta)}{\sin\theta} + \frac{\sin\theta}{\sin\theta}d\theta$$

$$= \frac{16}{\pi}\int_0^{\pi/2}(\cos 8\theta + \cos 6\theta + \cos 4\theta + \cos 2\theta + 1)d\theta + \frac{8}{\pi}\int_0^{\pi/2}d\theta$$

$$= \frac{16}{\pi}\left[\frac{\sin 8\theta}{8} + \frac{\sin 6\theta}{6} + \frac{\sin 4\theta}{4} + \frac{\sin 2\theta}{2}\right] + \frac{8}{\pi}[\theta]_0^{\pi/2} = 0 + \frac{8}{\pi}\times\left[\frac{\pi}{2} - 0\right] = 4$$